Ex 1 :

1) $f(-\sqrt{6}) = (-\sqrt{6})^2 = 6$

$f(1-\sqrt{2}) = (1-\sqrt{2})^2 = 1 - 2\sqrt{2} + 2 = \underline{3-2\sqrt{2}}.$

$f(10^{-2}) = (10^{-2})^2 = \underline{10^{-4}}$ [car $(10^n)^p = 10^{n\times p}$]

$f\left(\frac{7}{13}\right) = \left(\frac{7}{13}\right)^2 = \frac{7^2}{13^2} = \boxed{\frac{49}{169}}$

2) Antécédents de :

- $12 \longrightarrow x^2 = 12 \iff x = -\sqrt{12}$ ou $x = \sqrt{12}$

$\iff \underline{x = -2\sqrt{3}}$ ou $\underline{x = 2\sqrt{3}}$

$\sqrt{12} = \sqrt{4\times 3} = \sqrt{4}\times\sqrt{3} = 2\sqrt{3}$

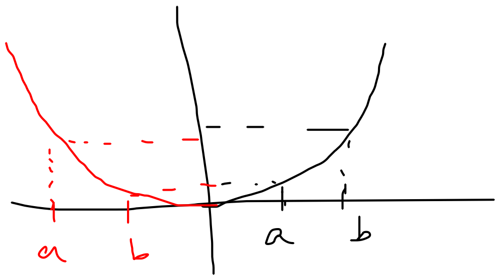
- $-4 \longrightarrow x^2 = -4$ impossible.

- $0 \longrightarrow x^2 = 0 \iff \underline{x = 0}$

- $\pi \longrightarrow x^2 = \pi \iff \underline{x = -\sqrt{\pi}}$ ou $\underline{x = \sqrt{\pi}}$

3) $A\left(-\frac{9}{2} ; 20\right)$. Si $A \in (\mathcal{P})$ alors $20 = \left(-\frac{9}{2}\right)^2$

soit $20 = \frac{81}{4} = 20{,}25$



Donc $A \notin (\mathcal{P})$.

4. Sans les calculer, comparer $f(\pi - 1)$ et $f(2,1)$. Comparer de même $f(-0,71)$ et $f\left(-\frac{2}{3}\right)$.

[redacted]

- $\pi - 1 \approx 3{,}14 - 1 = 2{,}14$ donc $0 < 2{,}1 < \pi - 1$

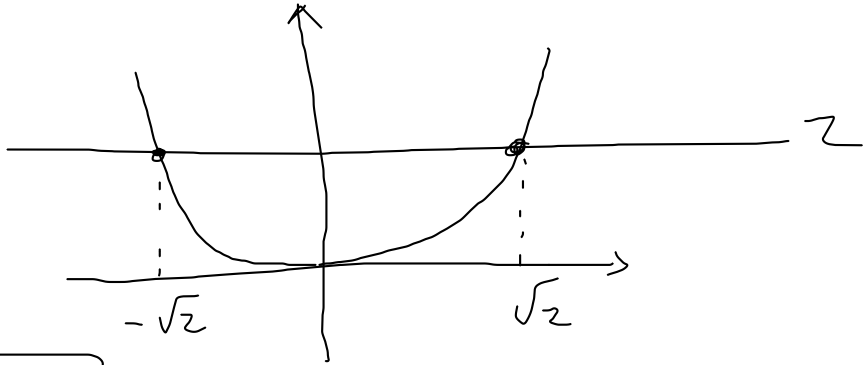
  donc $f(2{,}1) < f(\pi - 1)$ car f est croissante sur $[0 ; +\infty[$.

- $-\frac{2}{3} \approx -0{,}67$ donc $-0{,}71 < -0{,}67 < 0$

  donc $f(-0{,}71) > f\left(-\frac{2}{3}\right)$ car f est décroissante sur $]-\infty ; 0]$.

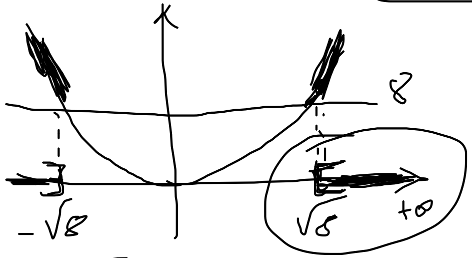
5) (a) $f(x) \leq 2$

$\iff \boxed{S = [-\sqrt{2} ; \sqrt{2}]}$



(b) $f(x) \geq 8$

$\iff \boxed{S =]-\infty ; -2\sqrt{2}] \cup [2\sqrt{2} ; +\infty[}$



$= -2\sqrt{2}$

(c) $f(x) \leq -1 \iff S = \emptyset$ car $x^2 \geq 0$ sur $\mathbb{R}$.

(d) $f(x) \geq 4 \iff S =]-\infty ; -2] \cup [2 ; +\infty[$

$\boxed{a > 0}$

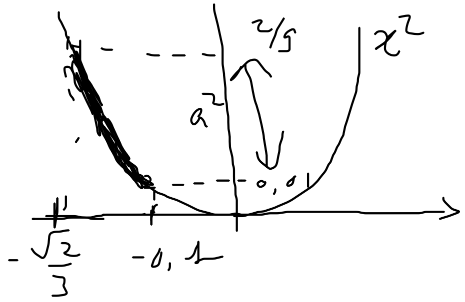
$f(x) \geq a \iff S =]-\infty ; -\sqrt{a}] \cup [\sqrt{a} ; +\infty[$

$f(x) \leq a \iff S = [-\sqrt{a} ; +\sqrt{a}]$

5) (a) $-\frac{\sqrt{2}}{3} \leq a \leq -0,1$

$\implies (-0,1)^2 \geq a^2 \geq \left(-\frac{\sqrt{2}}{3}\right)^2$ car f est décroissante

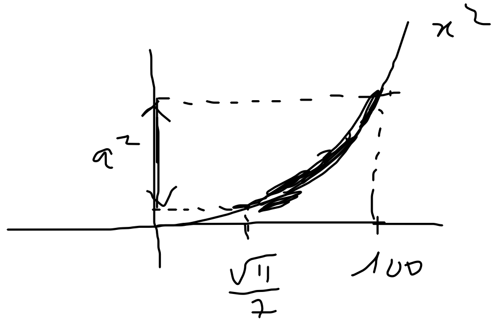
$\implies \boxed{0,01 \leq a^2 \leq \frac{2}{9}}$



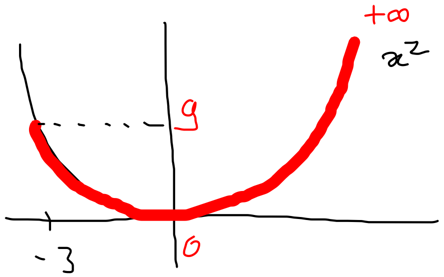
(b) $\frac{\sqrt{11}}{7} \leq a \leq 100$

$\implies \left(\frac{\sqrt{11}}{7}\right)^2 \leq a^2 \leq 100^2$

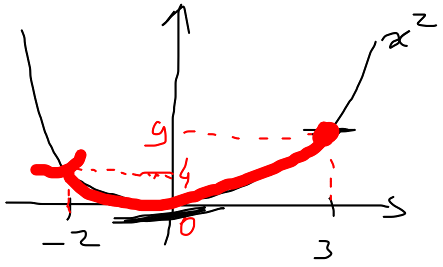
$\implies \boxed{\frac{11}{49} \leq a^2 \leq 10000}$



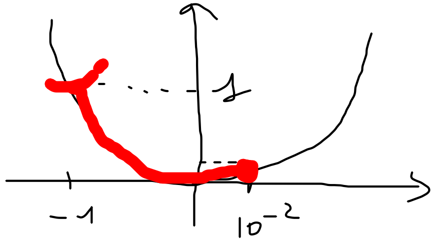
7) (a) $x \in [-3;+\infty[$; $x \geq -3$ donc $x^2 \in [0;+\infty[$



(b) $x \in]-2;3]$ donc $x^2 \in [0;9]$

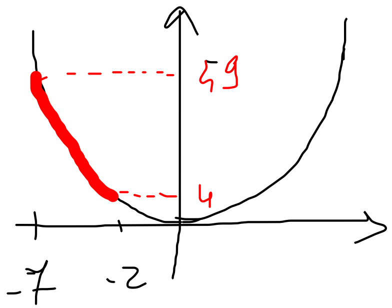


(c) $x \in]-1;10^{-2}]$ donc $x^2 \in [0;1[$



(d) $x \in [-7;-2]$ donc

$x^2 \in [4;49]$



Exercice 2 Soit $P(x) = 2x^2 - 2x - 4$. Déterminer les nombres a et β sachant que la forme canonique de P est $P(x) = a(x - \frac{1}{2})^2 + \beta$.

$$P(x) = a\left(x - \frac{1}{2}\right)^2 + \beta$$

$$= a\left(x^2 - x + \frac{1}{4}\right) + \beta$$

$$= \boxed{a}\,x^2 - \boxed{a}\,x + \boxed{\frac{1}{4}a + \beta}$$

$$= \boxed{2}\,x^2 - \boxed{2}\,x \boxed{- 4}$$

$-\frac{8}{2}$

Donc $\begin{cases} a = 2 \\ \frac{1}{4}a + \beta = -4 \end{cases}$ soit $\frac{1}{4} \times 2 + \beta = -4 \iff \frac{1}{2} + \beta = -4 \iff \beta = -4 - \frac{1}{2} = -\frac{9}{2}$

Donc $\boxed{P(x) = 2\left(x - \frac{1}{2}\right)^2 - \frac{9}{2}}$

↳ $S\left(\frac{1}{2} ; -\frac{9}{2}\right)$