Stéphane PASQUET, 21 novembre 2011

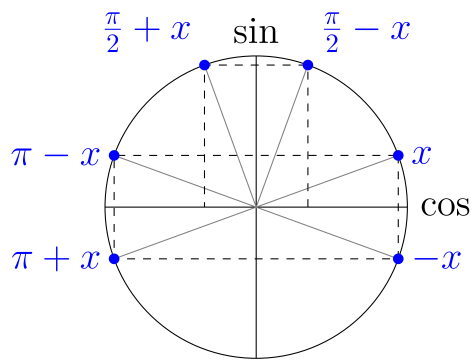
Trigonométrie

★ Angles associés

$$\begin{cases} \cos\left(\frac{\pi}{2}+x\right) = -\sin x \\ \sin\left(\frac{\pi}{2}+x\right) = \cos x \end{cases}$$

$$\begin{cases} \cos\left(\pi - x\right) = -\cos x \\ \sin\left(\pi - x\right) = \sin x \end{cases}$$

$$\begin{cases} \cos\left(\pi + x\right) = -\cos x \\ \sin\left(\pi + x\right) = -\sin x \end{cases}$$



$$\begin{cases} \cos\left(\frac{\pi}{2}-x\right) = \sin x \\ \sin\left(\frac{\pi}{2}-x\right) = \cos x \end{cases}$$

$$\begin{cases} \cos\left(-x\right) = \cos x \\ \sin\left(-x\right) = -\sin x \end{cases}$$

★ Egalités remarquables

$$\cos^2 x + \sin^2 x = 1 \qquad ; \qquad 1 + \tan^2 x = \frac{1}{\cos^2 x}$$

★ Formules d'addition

$$\cos(a-b) = \cos a \cos b + \sin a \sin b \qquad \sin(a-b) = \sin a \cos b - \cos a \sin b$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b \qquad \sin(a+b) = \sin a \cos b + \cos a \sin b$$

$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan(a)\tan(b)} \qquad \tan(a+b) = \frac{\tan a + \tan b}{1 - \tan(a)\tan(b)}$$

★ Formules de duplication

$$\begin{aligned} \cos(2a) &= \cos^2 a - \sin^2 a \\ &= 2\cos^2 a - 1 \\ &= 1 - 2\sin^2 a \end{aligned} \qquad \sin(2a) = 2\sin(a)\cos(a) \qquad \tan(2a) = \frac{2\tan a}{1-\tan^2 a}$$

$$\cos(a)\cos(b) = \frac{1}{2}\left[\cos(a-b) + \cos(a+b)\right]$$

$$\cos(a)\sin(b) = \frac{1}{2}\left[\sin(a+b) - \sin(a-b)\right]$$

$$\sin(a)\sin(b) = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right]$$

★ Formules diverses

$$\cos p + \cos q = 2\cos\left(\frac{p+q}{2}\right)\cos\left(\frac{p-q}{2}\right) \qquad \cos p - \cos q = -2\sin\left(\frac{p+q}{2}\right)\sin\left(\frac{p-q}{2}\right)$$

$$\sin p + \sin q = 2\sin\left(\frac{p+q}{2}\right)\cos\left(\frac{p-q}{2}\right) \qquad \sin p - \sin q = 2\cos\left(\frac{p+q}{2}\right)\sin\left(\frac{p-q}{2}\right)$$

★ Equations trigonométriques

$$\cos a = \cos b \iff \left(\begin{array}{l} a = b + 2k\pi,\ \ k \in \mathbb{Z} \\ ou \\ a = -b + 2k\pi,\ \ k \in \mathbb{Z} \end{array} \right.$$

$$\sin a = \sin b \iff \left(\begin{array}{l} a = b + 2k\pi,\ \ k \in \mathbb{Z} \\ ou \\ a = \pi - b + 2k\pi,\ \ k \in \mathbb{Z} \end{array} \right.$$

$$\tan a = \tan b \iff a = b + k\pi,\ \ k \in \mathbb{Z}$$

★ Représentations paramétriques

En posant $t = \tan\left(\frac{a}{2}\right)$, on a :

$$\cos a = \frac{1-t^2}{1+t^2} \quad ; \quad \sin a = \frac{2t}{1+t^2} \quad ; \quad \tan a = \frac{2t}{1-t^2}$$